

Interpreting the Predictions of Prediction Markets

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Abstract

Participants in prediction markets such as the Iowa Electronic Markets trade all-or-nothing contracts that pay a dollar if and only if specified future events occur. Researchers engaged in empirical study of prediction markets have argued broadly that equilibrium prices of the contracts traded are “market probabilities” that the specified events will occur. This paper shows that if traders are risk-neutral price takers with heterogeneous beliefs, the price of a contract in a prediction market reveals nothing about the dispersion of traders’ beliefs and partially identifies the central tendency of beliefs. Most persons have beliefs higher than price when price is above 0.5, and most have beliefs lower than price when price is below 0.5. The mean belief of traders lies in an interval whose midpoint is the equilibrium price. These findings persist even if traders use price data to revise their beliefs in plausible ways.

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1. Introduction

Prediction markets are futures markets “run for the primary purpose of using the information content in market values to make predictions about specific future events” (Berg and Rietz, 2003, p. 79). The operation since 1988 of the Iowa Electronic Markets (IEM) has stimulated considerable interest in such markets. The Iowa Political Markets, which trade futures contracts on U. S. election outcomes, have become especially well-known (Forsythe *et al.*, 1992). During each recent presidential campaign, market participants have been able to trade “all-or-nothing” contracts which pay one dollar if a specified candidate wins the popular vote and nothing otherwise. Researchers associated with the IEM have interpreted the equilibrium price of such a contract as a market-generated probability that the specified candidate will win the popular vote (Berg and Rietz, 2003; Berg, Nelson, and Rietz, 2003).¹

Prediction markets have attracted substantial public attention recently. In late 2002 and early 2003, the firm Tradesports.com enabled online trade in “Saddam Securities,” all-or-nothing futures contracts for the event that “Saddam Hussein is not President/Leader of Iraq” by a specified date.² Leigh, Wolfers, and Zitzewitz (2003) have interpreted the price of a Saddam Security as a market-generated probability of war with Iraq prior to the specified date.

Prediction markets achieved some notoriety in the summer of 2003, when, following public criticism, the Defense Advanced Research Projects Agency (DARPA) of the U.S. Department of Defense terminated development of its “Future Markets Applied to Prediction” (FutureMAP) program (see Carl Hulse, *The New York Times*, July 29, 2003, page A1). The DARPA News Release announcing cancellation of FutureMAP

¹ The Iowa Political Markets have also offered linear vote-share contracts which pay \$1 times the percentage of votes received by a candidate. In this case, researchers have interpreted the price of a contract as a market-generated best point prediction of the vote share that the candidate will receive.

² The website www.tradesports.com offers online trade in all-or-nothing contracts for many political, financial, and sporting events. An April 1, 2003 BBC report “Markets hold key to Saddam’s survival” illustrates the press coverage of Saddam Securities. See <http://news.bbc.co.uk/2/hi/business/2906143.stm>.

motivated the program as follows:³ “The FutureMAP research project was meant to explore the power of futures markets to predict and thereby prevent terrorist attacks. Futures markets have proven themselves to be good at predicting such things as elections results; they are often better than expert opinions.”

What is the logical basis for interpreting the price of an all-or-nothing futures contract as a market probability that the event will occur? Researchers engaged in empirical study of prediction markets have been uncomfortably vague. Forsythe *et al.* (1992) and Berg, Nelson, and Rietz (2003) refer to Hayek (1945), who argued broadly that market prices aggregate information.⁴ Leigh, Wolfers, and Zitzewitz (2003) write (p. 2): “Markets aggregate opinions and, by requiring a trader to ‘put your money where your mouth is,’ they lessen the cheap-talk problem and create incentives for individuals to reveal their true beliefs.” These and other recent papers on prediction markets provide no formal analysis showing how such markets aggregate information or opinions.

This paper shows, under special assumptions that may constitute a best-case scenario, what prices in prediction markets reveal about the beliefs of traders. Consider a market offering all-or-nothing contracts on the occurrence of a binary event; one contract pays a dollar if event m occurs and the other pays a dollar if the contrary event $n \equiv (\text{not } m)$ occurs. Let the prices of these contracts be π_m and π_n . Suppose that a population J with heterogeneous beliefs participates in this market. Each person $j \in J$ has a fixed trading budget of y_j dollars and a subjective probability q_{jm} that event m will occur; thus, $q_{jn} = 1 - q_{jm}$. Let $P(q_m, y)$ denote the cross-sectional distribution of beliefs and budgets. Assume that the distribution of beliefs is continuous and that budgets are statistically independent of beliefs. Finally, assume that persons are price takers and maximize the subjective expected value of their contracts.

³ See www.darpa.mil/body/newsitems/pdf/futuremaprelease2.pdf. The DARPA assertion of the power of future markets to predict election results presumably refers to the conclusions of researchers associated with the IEM, who have examined the predictive accuracy of the Iowa Political Markets (Berg, Nelson, and Rietz, 2003).

⁴ Hayek put it this way (p. 526): “The mere fact that there is one price for a commodity . . . brings about the solution which . . . might have been arrived at by one single mind possessing all the information which is in fact dispersed among all the people involved in the process.”

Section 2 shows that, under these assumptions, the unique equilibrium price for contract m in a market such as the IEM solves the equation

$$(1) \quad \pi_m = P(q_m > \pi_m).$$

The mean belief of the population of traders can take any value in the open interval below:

$$(2) \quad E(q_m) \in (\pi_m^2, 2\pi_m - \pi_m^2).$$

Thus, price does not generally equal the mean belief of traders, but π_m is the midpoint of an interval of width $2(\pi_m - \pi_m^2)$ that contains $E(q_m)$.

To illustrate, consider the price at the beginning of October 2002 of the Saddam Security paying off if Hussein is ousted by June 2003. Leigh, Wolfers, and Zitzewitz (2003, Figure 1) report that this price was 0.75. Under the maintained assumptions, this reveals that 75 percent of traders believed the probability of the event to be larger than 0.75. The mean subjective probability of Hussein's ouster lay somewhere in the range (0.5625, 0.935).

Equations (1) and (2) show what price reveals about beliefs if traders are risk-neutral with fixed beliefs and with budgets that are statistically independent of beliefs. These assumptions may or may not be realistic, but they provide a simple baseline for efforts to study more complex situations. One possibility is that, instead of beliefs being fixed, traders may use price data to revise their expectations. Thus, suppose that person j holds prior subjective probability q_{jm} when the market opens and revises this belief to $q_{jm}(\pi_m)$ after observing trades take place at price π_m . Section 3 shows that such revisions to beliefs do not change the equilibrium if prior and posterior beliefs bear the same ordinal relation to price. In particular, equation (1) continues to hold if each posterior belief $q_{jm}(\pi_m)$ is a weighted average of price π_m and the prior belief q_{jm} .

The concluding Section 4 observes that the information about beliefs contained in equations (1) and (2) is fragile. If budgets are not statistically independent of beliefs, prices in prediction markets are determined by the joint distribution of expectations and budgets rather than by the distribution of expectations alone. If traders are risk averse, price reflects their risk preferences as well as their expectations and budgets. For these and other reasons, I suggest that direct measurement of expectations may be preferable to inference on expectations from prices in prediction markets.

Although there appears to be no precedent study of the logic of price determination in prediction markets, relevant analysis has been performed in studies of *pari-mutuel* betting on horse races. Under behavioral assumptions similar to those used here, Eisenberg and Gale (1959) demonstrated the existence and uniqueness of equilibrium prices. Considering races with two horses, Ali (1977) reported equation (1) and suggested that this may explain the “favorite-longshot bias,” where horses with high equilibrium prices (i.e., favorites) empirically tend to win more often and those with low prices (i.e., longshots) tend to win less often than they should if their prices are interpreted as market probabilities of race outcomes.⁵ Brown and Lin (2003) shed further light on the favorite-longshot bias, showing that if the expectations of bettors are distributed Dirichlet, then the equilibrium prices of favorites are lower than the mean subjective probabilities that bettors hold for these horses to win.

⁵ Equation (1) here corresponds to Ali(1977), page 810, equation (4). His derivation rests on stronger assumptions than those used here; in particular, he assumed that all bettors wager the same amount. The Ali article is primarily an empirical analysis of *pari-mutuel* betting. He utilized extensive data on race outcomes to demonstrate the favorite-longshot bias, and he attempted to infer from these data the risk preferences of a representative bettor.

2. Beliefs and Prices in an Idealized Prediction Market

This section proves equations (1) and (2) under the assumptions of Section 1. Without loss of generality, let the unit interval index the members of J . Let (π_m, π_n) be any price vector such that $\pi_n = 1 - \pi_m$. It will be shown below that the equilibrium price vector satisfies this “no-arbitrage” condition.

Consider a person with trading budget y who places subjective probability q_m on occurrence of event m . Price taking and maximization of subjective expected value imply that this person invests his entire budget in contract m if $q_m > \pi_m$ and in contract n if $q_m < \pi_m$. Thus, the person purchases y/π_m units of contract m if $q_m > \pi_m$ and y/π_n units of contract n if $q_m < \pi_m$. All portfolios are optimal if $q_m = \pi_m$, but the assumption that $P(q_m)$ is continuous implies that this equality almost never occurs among the members of J . Hence, aggregating individual behavior across the population, the market demands for contracts m and n are $(1/\pi_m) \cdot E\{y \cdot 1[q_m > \pi_m]\}$ and $(1/\pi_n) \cdot E\{y \cdot 1[q_m < \pi_m]\}$ respectively.

In the IEM, a person with budget y is given an endowment of y units of contract m and y units of contract n . Hence, the market supply of each contract is $E(y)$. Equilibrium requires that the market demand for each contract equal this common supply. Thus, (π_m, π_n) is an equilibrium price vector if and only if

$$(3) \quad E(y) = (1/\pi_m) \cdot E\{y \cdot 1[q_m > \pi_m]\} = (1/\pi_n) \cdot E\{y \cdot 1[q_m < \pi_m]\}.$$

If y and q_m are statistically independent, then $E\{y \cdot 1[q_m > \pi_m]\} = E(y) \cdot P(q_m > \pi_m)$ and $E\{y \cdot 1[q_m < \pi_m]\} = E(y) \cdot P(q_m < \pi_m)$. Hence, equation (3) reduces to

$$(4a) \quad \pi_m = P(q_m > \pi_m),$$

$$(4b) \quad \pi_n = P(q_m < \pi_m).$$

Equality (4a) is equation (1). Continuity of $P(q_m)$ implies that $P(q_m < \pi_m) = 1 - P(q_m > \pi_m)$, so (4a) and (4b) yield the no-arbitrage condition. Equation (1) has a unique solution because $P(q_m > \pi_m)$ is a continuous function of π_m that decreases in value from 1 to 0 as π_m increases from 0 to 1.⁶

The bound (2) on $E(q_m)$ follows from (1). To obtain the upper bound, observe that the distribution that satisfies (1) and has the largest possible mean is the two-point distribution with $P(q_m = \pi_m) = 1 - \pi_m$ and $P(q_m = 1) = \pi_m$. This distribution has mean $2\pi_m - \pi_m^2$. The assumption that $P(q_m)$ is continuous implies that this two-point distribution is not feasible. However, for $0 < \delta < \min(\pi_m, 1 - \pi_m)$, all continuous distributions placing probabilities $1 - \pi_m$ and π_m on the intervals $(\pi_m - \delta, \pi_m)$ and $(1 - \delta, 1)$ satisfy (1). Such distributions have means that are smaller than $2\pi_m - \pi_m^2$ but that approach this value as $\delta \rightarrow 0$.

To obtain the lower bound, replace (1) by the equation

$$(1') \quad \pi_m = P(q_m \geq \pi_m),$$

which is equivalent to (1) if $P(q_m)$ is continuous. The distribution that satisfies (1') and has the smallest possible mean is the two-point distribution with $P(q_m = 0) = 1 - \pi_m$ and $P(q_m = \pi_m) = \pi_m$. This distribution has mean π_m^2 , but is not feasible. However, for $0 < \delta < \min(\pi_m, 1 - \pi_m)$, all continuous distributions placing probabilities $1 - \pi_m$ and π_m on the intervals $(0, \delta)$ and $(\pi_m, \pi_m + \delta)$ satisfy (1). Such distributions have means that are larger than π_m^2 but that approach this value as $\delta \rightarrow 0$.

It remains to show that $E(q_m)$ can take any value in the interval $(\pi_m^2, 2\pi_m - \pi_m^2)$. The argument above shows that, for small positive values of λ , there exist continuous distributions that satisfy (1) and that have means $\pi_m^2 + \lambda$ and $2\pi_m - \pi_m^2 - \lambda$. Let $0 < \alpha < 1$ and consider a $(1 - \alpha, \alpha)$ mixture of such distributions. The

⁶ Pari-mutuel markets differ institutionally from the IEM, but the same equilibrium condition holds if the “track take” is zero; that is, if the track returns all money to the bettors. Then the price of a “win” bet on a horse is the fraction of the total betting pool that is wagered on this horse. Consider a race with two horses, say m and n . In the notation of this paper, the aggregate amount bet on horse m is $E\{y \cdot 1[q_m > \pi_m]\}$ and the betting pool is $E(y)$. Hence, the equilibrium price of a bet on horse m solves the equation $\pi_m = E\{y \cdot 1[q_m > \pi_m]\}/E(y)$. If y is statistically independent of q_m , this reduces to (1).

mixture is continuous, satisfies (1), and has mean $(1 - \alpha)(\pi_m^2 + \lambda) + \alpha(2\pi_m - \pi_m^2 - \lambda)$. Considering all $\alpha \in (0, 1)$ and letting $\lambda \rightarrow 0$ proves the result.

3. Using Price Data to Revise Expectations

Section 2 assumed that persons hold fixed beliefs about the likelihood of event m . A prominent theme of theoretical research in information economics has been that prices may reveal private information held by market participants. Extrapolating from this research to prediction markets, one might conjecture that traders in prediction markets use price data to revise their expectations.

Whether and how traders in prediction markets use price data to revise their expectations is an open empirical question that cannot be addressed here. Instead, the analysis in this section makes two assumptions about the posterior beliefs that traders hold. First, trading budgets are statistically independent of posterior beliefs. Second, each person j 's posterior belief $q_{jm}(\pi_m)$ bears the same ordinal relationship to price as did his prior belief q_{jm} ; that is,

$$(5) \quad \text{sgn}[q_{jm}(\pi_m) - \pi_m] = \text{sgn}[q_{jm} - \pi_m].$$

These assumptions do not mandate a particular rule for revision of expectations, but are consistent with many possible rules. For example, the assumptions hold if $q_{jm}(\pi_m)$ is a weighted average of π_m and q_{jm} of the form $q_{jm}(\pi_m) = \theta_j q_{jm} + (1 - \theta_j)\pi_m$, where $\theta_j \in (0, 1]$, and if budgets are statistically independent of (q_m, θ) .

Repeating the derivation of Section 2, (π_m, π_n) is an equilibrium price vector if and only if

$$(6) \quad E(y) = (1/\pi_m) \cdot E\{y \cdot 1[q_m(\pi_m) > \pi_m]\} = (1/\pi_n) \cdot E\{y \cdot 1[q_m(\pi_m) < \pi_m]\}.$$

Statistical independence of budgets and posterior beliefs implies that (6) reduces to

$$(7a) \quad \pi_m = P[q_m(\pi_m) > \pi_m],$$

$$(7b) \quad \pi_n = P[q_m(\pi_m) < \pi_m].$$

Condition (5) implies that

$$(8a) \quad P[q_m(\pi_m) > \pi_m] = P(q_m > \pi_m),$$

$$(8b) \quad P[q_m(\pi_m) < \pi_m] = P(q_m < \pi_m).$$

Combining (7a)-(7b) and (8a)-(8b) yields (4a)-(4b). Hence, the equilibrium price using posterior beliefs is the same as the one using prior beliefs.

4. Discussion

The above analysis refutes the notion that prices in prediction markets are “market probabilities.” Under the maintained assumptions, the price of an all-or-nothing futures contract does not equal the mean, median, or any other measure of the central tendency of traders’ beliefs. Instead, price solves equation (1). Thus, most persons have beliefs higher than price when price is above 0.5, and most have beliefs lower than price when price is below 0.5.

The analysis shows that price reveals nothing about the dispersion of traders’ beliefs. Equation (1) can hold if beliefs are very homogeneous, with all of the probability mass of $P(q_m)$ concentrated in a small neighborhood of the value π_m . It can also hold if beliefs are highly heterogeneous, with some traders giving

event m probability close to zero and the rest giving it probability close to one. The invariance of equilibrium to changes in the dispersion of beliefs underlies the finding of Section 3 that the equilibrium stays the same if traders use price data to revise beliefs. If posterior beliefs satisfy equation (5), the distributions of prior and posterior beliefs may have different dispersions but equation (1) holds both a priori and a posteriori.

The analysis shows that price does reveal something about the central tendency of traders' beliefs. Equation (2) shows that $E(q_m)$ lies in the interval $(\pi_m^2, 2\pi_m - \pi_m^2)$. This interval has midpoint π_m , shifts rightward as π_m increases, and has width $2\pi_m(1 - \pi_m)$. Thus, prices near zero or one are very informative about the mean beliefs of traders, but prices near 0.5 are much less informative.

Although the simple model studied here implies that price reveals something about beliefs, it does not support the *efficient markets hypothesis* that price is a sufficient statistic for all private information held by traders. Demonstrations that markets can be efficient, such as Theorem 1 of Grossman (1976), have assumed that traders initially view the world in the same way (i. e., have *common priors*), observe different data (or *private signals*) about this world, use Bayes rule to integrate data and initial beliefs, trade based on their posterior beliefs, and further revise their beliefs after observing the price at which trades take place. Theorists have assumed specific forms for the common prior (e. g., a normal prior distribution for the value of an asset) and for the private signals that traders observe (e. g., the signal equals the value of the asset plus normal white noise). Moreover, they have assumed that all of this structure is common knowledge. The present analysis of all-or-nothing prediction markets maintains none of these assumptions; hence, it is no surprise that efficient markets do not emerge here.

Application of equations (1) and (2) to real prediction markets should be cautious, because these results may not be robust to relaxation of the assumptions used to derive them. Among the many ways to relax the assumptions, here are two. First, suppose that budgets and beliefs may be statistically dependent. Then equation (3) shows that price depends on the conjunction of traders' budgets and beliefs rather than on their beliefs alone. Equilibrium condition (1) does not hold as stated but does hold if one redefines J to be

the population of dollars traded rather than the population of traders. Then equation (2) gives a bound on budget-weighted mean beliefs.

Now suppose that traders may be risk averse rather than maximizers of expected value. Should a risk-averse person invest in a prediction market, he will invest in contract m only if $q_m > \pi_m$. However, such a person may choose not to invest his entire trading budget but rather an amount that depends on price and on his beliefs and risk preferences. Hence, price solves the equation

$$(9) \quad \pi_m = E[x(q_m, y, r; \pi_m) | q_m > \pi_m] \cdot P(q_m > \pi_m) / E[x(q_m, y, r; \pi_m)],$$

where $x(q_m, y, r; \pi_m)$ is the amount that a person with beliefs q_m , budget y , and risk preferences r invests at price π_m . Thus, price depends on the joint distribution of (beliefs, budgets, risk preferences).

If one wants to predict a future event, an alternative to study of prices in prediction markets is direct measurement of the expectations of persons who may have informed beliefs about this event. Until recently, economists largely refrained from measurement of expectations.⁷ In the past decade, however, economists have amassed considerable experience eliciting from survey respondents probabilistic expectations of diverse events, from mortality (Hurd and McGarry, 1995) to job loss (Dominitz and Manski, 1997) to stock market returns (Dominitz and Manski, 2004); Manski (2004) reviews this literature. Whereas prediction markets only provide data on persons who choose to trade, surveys enable measurement of the expectations of random samples of populations of interest. Whereas prediction markets at most provide information on the central tendency of beliefs, measurement of subjective probabilities can reveal the dispersion of beliefs as well. A possible advantage of prediction markets relative to recent work eliciting expectations is that prediction

⁷ The expectations surveys which did draw some attention, such as the Livingston Panel and the Survey of Professional Forecasters, only ask respondents for point predictions of future events rather than for the subjective probabilities that they hold. I have shown elsewhere that point predictions at most yield a bound on the mean beliefs of respondents; indeed, this bound is related to equation (2). See Manski (1990), Section 3.3.

markets provide monetary incentives for accuracy, whereas surveys of expectations have not done so. To address this concern, survey researchers could provide incentives through the use of proper scoring rules, which reward respondents for accurate probabilistic prediction (e.g., Shuford, Albert, and Massengill, 1966; Savage, 1971).

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